Journal of Engineering Physics and Thermophysics, Vol. 83, No. 1, 2010

ACCOUNT FOR HEAT TRANSFER BETWEEN ELEMENTS OF A PLANE PARALLEL STRIP–FOUNDATION FRICTION UNIT

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An analytical solution of the boundary-value heat conduction problem for a tribosystem consisting of a semi-infinite foundation and a plane-parallel strip sliding at a constant velocity along the foundation surface is obtained. The thermal contact between the strip and the foundation is partial. The asymptotics of the solution for low and high values of time have been found. For the materials of the friction pair "metal ceramics strip–pig iron foundation" the influence of the coefficient of thermal conductivity of the contact on temperature distribution was studied.

Keywords: *temperature, heat transfer, thermal conductivity of the contact, thermal problem of friction.*

Introduction. Thermal conditions on the friction surface of two elastic bodies were written for the first time in modern form by F. Ling [1], who was of the opinion that at each point in the region of contact the temperatures of the contacting bodies were the same, whereas the sum of the intensities of heat fluxes directed from the friction surface inside each body was equal to the specific power of friction (ideal or full thermal contact). Such an approach to the statement of contact problems with account for frictional heat generation presupposes the dependence of temperature only on the thermophysical properties of the friction unit materials, cooling conditions, and friction power.

It is known that the processes of friction, wear, and heat generation depend greatly on the topological parameters of the contacting surfaces and on the characteristics of the so-called "third body," that is, a thin near-surface strip whose physicomechanical properties differ from the properties of the contacting bodies [2, 3]. The thermal contact of bodies at which the temperature on the frictional surface undergoes a jump is called partial (nonideal) [4, 5]. The solution of a contact thermal problem of friction for two semi-infinite bodies under such thermal boundary conditions was obtained in [6, 7] for the case of their uniform sliding and in [8] for sliding with constant retardation.

The frictional heat generation in the tribosystem "plane-parallel strip–foundation," provided there is equality of temperatures on the contact surface, was investigated in [9–12]. The aim of the present work is to obtain an analytical solution of a thermal contact problem for this very tribosystem with account for heat transfer between the contacting surfaces.

Statement of the Problem. Let a plane-parallel strip of thickness d and a semi-infinite foundation (semispace) be compressed by a normal loading of constant intensity p_0 applied to the external surface of the strip and at infinity in the semispace (Fig. 1). At the initial moment $t = 0$ the strip begins to slide over the semispace surface with a constant velocity V in the positive direction of the y axis. Due to the friction on the contact surface $z = 0$ heat is generated, and the tribosystem is heated. It is assumed that there is transfer between the contacting surfaces of the strip and foundation with a constant coefficient of thermal conductivity of the contact h, and the sum of the intensities of heat fluxes directed from the frictional surface inside each of the bodies is equal to the specific power of friction. The temperature on the external surface of the strip $z = d$ is zero. The wear of the rubbing surfaces is neglected.

Under the assumptions made, the distributions of nonstationary temperatures in the strip $T_s(z, t)$ and foundation $T_f(z, t)$ is given by the solution of the boundary-value problem of heat conduction:

$$\frac{\partial^2 T_s(z, t)}{\partial z^2} = \frac{1}{k_s}\frac{\partial T_s(z, t)}{\partial t}, \quad 0 < z < d, \quad t > 0; \tag{1}$$

Bialystok Polytechnic Institute, 45 C Veiska Str., Bialystok. 15–351, Poland. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 83, No. 1, pp. 38–46, January–February, 2010. Original article submitted December 16, 2008; revision submitted June 1, 2009.

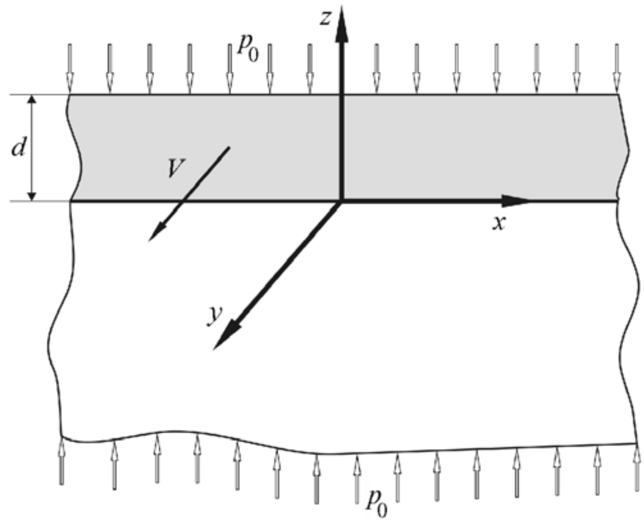

Fig. 1. Schematic diagram of the strip and foundation.

$$\frac{\partial^2 T_f(z,t)}{\partial z^2} = \frac{1}{k_f}\frac{\partial T_f(z,t)}{\partial t}, \quad -\infty < z < 0, \quad t > 0; \tag{2}$$

$$K_f \frac{\partial T_f}{\partial z}\bigg|_{z=0-} - K_s \frac{\partial T_s}{\partial z}\bigg|_{z=0+} = q \equiv fVp_0, \quad t > 0; \tag{3}$$

$$K_f \frac{\partial T_f}{\partial z}\bigg|_{z=0-} + K_s \frac{\partial T_s}{\partial z}\bigg|_{z=0+} = h\,[T_s(0,t) - T_f(0,t)], \quad t > 0; \tag{4}$$

$$T_s(d,t) = 0, \quad t > 0; \tag{5}$$

$$T_f(z,t) \to 0, \quad z \to -\infty, \quad t > 0; \tag{6}$$

$$T_s(z,0) = 0, \quad 0 \le z \le d, \quad T_f(z,0) = 0, \quad -\infty < z \le 0. \tag{7}$$

The methods of determining the thermal conductivity coefficient of the contact h can be found in [13]. We now pass to dimensionless quantities and parameters:

$$\zeta = \frac{z}{d}, \quad \tau = \frac{k_s t}{d^2}, \quad K^* = \frac{K_f}{K_s}, \quad k^* = \frac{k_f}{k_s}, \quad \mathrm{Bi} = \frac{hd}{K_s}, \quad T_0 = \frac{qd}{K_s}, \quad T_s^* = \frac{T_s}{T_0}, \quad T_f^* = \frac{T_f}{T_0}, \tag{8}$$

the boundary-value problem (1)–(7) is written in the form

$$\frac{\partial^2 T_s^*(\zeta,\tau)}{\partial \zeta^2} = \frac{\partial T_s^*(\zeta,\tau)}{\partial \tau}, \quad 0 < \zeta < 1, \quad \tau > 0; \tag{9}$$

$$\frac{\partial^2 T_f^*(\zeta,\tau)}{\partial \zeta^2} = \frac{1}{k^*}\frac{\partial T_f^*(\zeta,\tau)}{\partial \tau}, \quad -\infty < \zeta < 0, \quad \tau > 0; \tag{10}$$

$$K^* \left.\frac{\partial T_f^*}{\partial \zeta}\right|_{\zeta=0-} - \left.\frac{\partial T_s^*}{\partial \zeta}\right|_{\zeta=0+} = 1, \quad \tau > 0; \tag{11}$$

$$K^* \left.\frac{\partial T_f^*}{\partial \zeta}\right|_{\zeta=0-} + \left.\frac{\partial T_s^*}{\partial \zeta}\right|_{\zeta=0+} = \mathrm{Bi}\,[T_s^*(0,\tau) - T_f^*(0,\tau)], \quad \tau > 0; \tag{12}$$

$$T_s^*(1,\tau) = 0, \quad \tau > 0; \tag{13}$$

$$T_f^*(\zeta,\tau) \to 0, \quad \zeta \to -\infty, \quad \tau > 0; \tag{14}$$

$$T_s^*(\zeta,0) = 0, \quad 0 \le \zeta \le 1, \quad T_f^*(\zeta,0) = 0, \quad -\infty < \zeta \le 0. \tag{15}$$

Solution of the Problem. The solution of the thermal problem of friction (9)–(15) in the space of inverse transforms of the Laplace integral transformation [14]

$$L\,[T_{s,f}^*(\zeta,\tau); p] \equiv \overline{T}_{s,f}^*(\zeta,p) = \int_0^\infty T_{s,f}^*(\zeta,\tau)\exp(-p\tau)\,d\tau \tag{16}$$

has the form

$$\overline{T}_s^*(\zeta,p) = \frac{\Delta_s(\zeta,p)}{p\Delta(p)}, \quad 0 \le \zeta \le 1; \tag{17}$$

$$\overline{T}_f^*(\zeta,p) = \frac{\Delta_f(\zeta,p)}{p\Delta(p)}, \quad -\infty < \zeta \le 0, \tag{18}$$

where

$$\Delta_s(\zeta,p) = \left(\varepsilon + \frac{\mathrm{Bi}}{\sqrt{p}}\right)\sinh\,[(1-\zeta)\sqrt{p}\,], \quad 0 \le \zeta \le 1; \tag{19}$$

$$\Delta_f(\zeta,p) = \left[\cosh\sqrt{p} + \frac{\mathrm{Bi}}{\sqrt{p}}\sinh\sqrt{p}\right]\exp\left(\zeta\sqrt{\frac{p}{k^*}}\right), \quad -\infty < \zeta \le 0; \tag{20}$$

$$\Delta(p) = \varepsilon\,\mathrm{Bi}\sinh\sqrt{p} + (2\varepsilon\sqrt{p} + \mathrm{Bi})\cosh\sqrt{p}; \tag{21}$$

$\varepsilon = K^* / \sqrt{k^*}$ is the thermal activity coefficient of the foundation material relative to the strip material (15). Passing in Eqs. (17)–(21) to inverse transforms by integration in the plane of the complex variable p along a closed circuit with a cut in the negative direction of the real axis, we find the dimensionless temperature in the strip and foundation:

$$T_s^* (\zeta, \tau) = 1 - \zeta - \frac{2}{\pi} \int_0^\infty F(x)\, G_s(\zeta, x) \exp(-x^2 \tau)\, dx\,, \quad 0 \le \zeta \le 1\,, \quad \tau \ge 0\,; \tag{22}$$

$$T_f^* (\zeta, \tau) = 1 + \frac{1}{\mathrm{Bi}} - \frac{2}{\pi} \int_0^\infty F(x)\, G_f(\zeta, x) \exp(-x^2 \tau)\, dx\,, \quad -\infty < \zeta \le 0\,, \quad \tau \ge 0\,; \tag{23}$$

$$F(x) = \frac{\cos x + \mathrm{Bi}\, x^{-1} \sin x}{(\mathrm{Bi} \cos x)^2 + \varepsilon^2 (\mathrm{Bi} \sin x + 2x \cos x)^2}\,; \tag{24}$$

$$G_s(\zeta, x) = \varepsilon\, \mathrm{Bi}\, x^{-1} \sin[(1 - \zeta)\, x]\,, \quad 0 \le \zeta \le 1\,; \tag{25}$$

$$G_f(\zeta, x) = \varepsilon\, (\mathrm{Bi}\, x^{-1} \sin x + 2 \cos x) \cos\left(\frac{\zeta x}{\sqrt{k^*}}\right) - \mathrm{Bi}\, x^{-1} \cos x \sin\left(\frac{\zeta x}{\sqrt{k^*}}\right), \quad -\infty < \zeta \le 0\,. \tag{26}$$

It follows from relations (22) and (25) that boundary condition (13) on the external surface of a strip is satisfied.

Of greatest interest is the maximum temperature attained on the friction surface. To find this temperature, we use solutions (22) and (23) at $\zeta = 0$ and, according to Eqs. (25) and (26), at integrand functions

$$G_s(0, x) = \varepsilon\, \mathrm{Bi}\, x^{-1} \sin x\,, \quad G_f(0, x) = \varepsilon\, (\mathrm{Bi}\, x^{-1} \sin x + 2 \cos x)\,. \tag{27}$$

The intensities of heat fluxes in the strip and foundation are equal to

$$q_s(z, t) \equiv -K_s \frac{\partial T_s(z, t)}{\partial z}\,, \quad 0 \le z \le d\,, \quad t \ge 0\,, \quad q_f(z, t) \equiv K_f \frac{\partial T_f(z, t)}{\partial z}\,, \quad -\infty < z \le 0\,, \quad t \ge 0\,,$$

or, with allowance for the notation (8),

$$q_s^* (\xi, \tau) \equiv \frac{q_s(z, t)}{q} = -\frac{\partial T_s^* (\zeta, \tau)}{\partial \zeta}\,, \quad 0 \le \zeta \le 1\,, \quad \tau \ge 0\,; \tag{28}$$

$$q_f^* (\zeta, \tau) \equiv \frac{q_f(z, t)}{q} = K^* \frac{\partial T_f^* (\zeta, \tau)}{\partial \zeta}\,, \quad -\infty < \zeta \le 0\,, \quad \tau \ge 0\,, \tag{29}$$

are found from solutions of (22)–(27) in the form

$$q_s^* (\zeta, \tau) = 1 - \frac{2\varepsilon}{\pi} \int_0^\infty F(x)\, Q_s(\zeta, x) \exp(-x^2 \tau)\, dx\,, \quad 0 \le \zeta \le 1\,, \quad \tau \ge 0\,; \tag{30}$$

$$q_f^*(\zeta, \tau) = \frac{2\varepsilon}{\pi} \int_0^\infty F(x)\, Q_f(\zeta, x) \exp(-x^2\tau)\, dx\,, \quad -\infty < \zeta \le 0\,, \quad \tau \ge 0\,; \tag{31}$$

$$Q_s(\zeta, x) = \mathrm{Bi} \cos[(1-\zeta)\, x]\,, \quad 0 \le \zeta \le 1\,; \tag{32}$$

$$Q_f(\zeta, x) = \varepsilon\,(\mathrm{Bi} \sin x + 2x \cos x) \sin\left(\frac{\zeta x}{\sqrt{k^*}}\right) + \mathrm{Bi} \cos x \cos\left(\frac{\zeta x}{\sqrt{k^*}}\right), \quad -\infty < \zeta \le 0\,, \tag{33}$$

where the integrand $F(x)$ has the form of (24).

At $\zeta = 0$ Eqs. (32) and (33) yield $Q_f(0, x) = Q_s(0, x) = \mathrm{Bi} \cos x$, and from relations (30) and (31) it follows that boundary condition (11) is satisfied. From solutions (22), (23) and (30), (31) we find the temperature jumps and heat flux intensities on the contact surface:

$$T_s^*(0, \tau) - T_f^*(0, \tau) = -\frac{1}{\mathrm{Bi}} + \frac{2}{\pi} \int_0^\infty F(x)\, [G_f(0, x) - G_s(0, x)] \exp(-x^2\tau)\, dx\,, \quad \tau \ge 0\,; \tag{34}$$

$$q_f^*(0, \tau) - q_s^*(0, \tau) = -1 + \frac{2\varepsilon}{\pi} \int_0^\infty F(x)\, [Q_f(0, x) + Q_s(0, x)] \exp(-x^2\tau)\, dx\,, \quad \tau \ge 0\,, \tag{35}$$

where, subject to Eqs. (27) and (32), (33),

$$G_f(0, x) - G_s(0, x) = 2\varepsilon \cos x\,, \quad Q_f(0, x) + Q_s(0, x) = 2\, \mathrm{Bi} \cos x\,. \tag{36}$$

Based on Eqs. (34)–(36), we conclude that boundary condition (12) is satisfied.

We find the dimensionless temperatures and heat flux intensities in the case of an ideal thermal contact of the strip with the foundation from Eqs. (22)–(26) and (30)–(33), passing in them to the limit $\mathrm{Bi} \to \infty$. As a result we obtain

$$F(x) = \frac{x^{-1} \sin x}{\cos^2 x + \varepsilon^2 \sin^2 x}\,; \tag{37}$$

$$G_s(\zeta, x) = \varepsilon x^{-1} \sin[(1-\zeta)\, x]\,, \quad Q_s(\zeta, x) = \cos[(1-\zeta)\, x]\,, \quad 0 \le \zeta \le 1\,; \tag{38}$$

$$G_f(\zeta, x) = \varepsilon x^{-1} \sin x \cos(\zeta x/\sqrt{k^*}) = x^{-1} \cos x \sin(\zeta x/\sqrt{k^*})\,, \quad -\infty < \zeta \le 0\,; \tag{39}$$

$$Q_f(\zeta, x) = \varepsilon \sin x \sin(\zeta x/\sqrt{k^*}) + \cos x \cos(\zeta x/\sqrt{k^*})\,, \quad -\infty < \zeta \le 0\,. \tag{40}$$

On the contact surface $\zeta = 0$ from Eqs. (37)–(40) we find

$$G_s(0, x) = G_f(0, x) = \varepsilon x^{-1} \sin x\,, \quad Q_s(0, x) = Q_f(0, x) = \cos x\,. \tag{41}$$

Equations (37)–(41) as the solutions of the thermal problem of friction in the case of a close thermal contact between the strip and foundation were obtained by us in [10, 11].

Asymptotic Solutions. At small values of dimensionless time (the Fourier number) the parameter of the Laplace integral transformation (16) $p \to \infty$ and the transformed solutions (17) and (18) take the form

$$\overline{T}_{\rm s}^{*}(\zeta, p) \cong \frac{\left(\varepsilon + \frac{\rm Bi}{\sqrt{p}}\right)\exp\left(-\zeta\sqrt{p}\right)}{2\varepsilon p\left(\alpha + \sqrt{p}\right)}, \quad 0 \le \zeta \le 1, \quad \overline{T}_{\rm f}^{*}(\zeta, p) \cong \frac{\left(\varepsilon + \frac{\rm Bi}{\sqrt{p}}\right)\exp\left(\zeta\sqrt{\frac{p}{k^{*}}}\right)}{2\varepsilon p\left(\alpha + \sqrt{p}\right)}, \quad -\infty < \zeta \le 0, \tag{42}$$

where

$$\alpha = \frac{(1+\varepsilon)}{2\varepsilon}\,{\rm Bi}\,. \tag{43}$$

With the use of the couplings [16]

$$L^{-1}\left[\frac{\alpha\exp\left(-|\zeta|\sqrt{p}\right)}{p\left(\alpha+\sqrt{p}\right)}; \tau\right] = {\rm erfc}\left(\frac{|\zeta|}{2\sqrt{\tau}}\right) - \exp\left(\alpha\,|\zeta| + \alpha^2\tau\right){\rm erfc}\left(\frac{|\zeta|}{2\sqrt{\tau}} + \alpha\sqrt{\tau}\right);$$

$$L^{-1}\left[\frac{\exp\left(-|\zeta|\sqrt{p}\right)}{p\sqrt{p}\left(\alpha+\sqrt{p}\right)}; \tau\right] = \frac{2}{\alpha}\sqrt{\frac{\tau}{\pi}}\exp\left(-\frac{\zeta^2}{4\pi}\right) - \left(\frac{|\zeta|}{\alpha} + \frac{1}{\alpha^2}\right){\rm erfc}\left(\frac{|\zeta|}{2\sqrt{\tau}}\right)$$

$$+ \frac{1}{\alpha^2}\exp\left(\alpha\,|\zeta| + \alpha^2\tau\right){\rm erfc}\left(\frac{|\zeta|}{2\sqrt{\tau}} + \alpha\sqrt{\tau}\right) \tag{44}$$

form relations (42) we find asymptotic, for small values of time ($0 \le \tau \le 1$), expressions for the strip and foundation temperatures:

$$T_{\rm s}^{*}(\zeta, \tau) \cong \frac{2\sqrt{\tau}}{1+\varepsilon}\,{\rm ierfc}\left(\frac{\zeta}{2\sqrt{\tau}}\right) - \frac{\lambda}{2\alpha}\left[{\rm erfc}\left(\frac{\zeta}{2\sqrt{\tau}}\right) - \exp\left(\alpha\,|\zeta| + \alpha^2\tau\right){\rm erfc}\left(\frac{\zeta}{2\sqrt{\tau}} + \alpha\sqrt{\tau}\right)\right], \quad 0 \le \zeta \le 1; \tag{45}$$

$$T_{\rm f}^{*}(\zeta, \tau) \cong \frac{2\sqrt{\tau}}{1+\varepsilon}\,{\rm ierfc}\left(\frac{|\zeta|}{2\sqrt{k^{*}\tau}}\right) + \frac{\lambda}{2\alpha\varepsilon}\left[{\rm erfc}\left(\frac{|\zeta|}{2\sqrt{k^{*}\tau}}\right) - \exp\left(\alpha\frac{|\zeta|}{\sqrt{k^{*}}} + \alpha^2\tau\right){\rm erfc}\left(\frac{|\zeta|}{2\sqrt{k^{*}\tau}} + \alpha\sqrt{\tau}\right)\right], \quad -\infty < \zeta \le 0, \tag{46}$$

where

$$\lambda = \frac{1-\varepsilon}{1+\varepsilon}\,. \tag{47}$$

Passing in Eqs. (45) and (46) to the limit $\tau \to 0$, we find that $T_{\rm s,f}^{*}(\zeta, 0) = 0$. This indicates the fulfillment of initial conditions (15). Assuming in Eqs. (45) and (46) that $\zeta = 0$, we determine the temperatures on the contact surface of the strip and foundation:

$$T_{\rm s}^{*}(0, \tau) \cong \frac{2}{1+\varepsilon}\sqrt{\frac{\tau}{\pi}} - \frac{\lambda}{2\alpha}\left[1 - \exp\left(\alpha^2\tau\right){\rm erfc}\left(\alpha\sqrt{\tau}\right)\right];$$

$$T_{\rm f}^{*}(0, \tau) \cong \frac{2}{1+\varepsilon}\sqrt{\frac{\tau}{\pi}} + \frac{\lambda}{2\alpha\varepsilon}\left[1 - \exp\left(\alpha^2\tau\right){\rm erfc}\left(\alpha\sqrt{\tau}\right)\right], \quad 0 \le \tau << 1. \tag{48}$$

Using the notation of (43) and (47), from Eqs. (48) we obtain

$$T_s^*(0,\tau) - T_f^*(0,\tau) = -\frac{\lambda}{\mathrm{Bi}}\,[1 - \exp(\alpha^2\tau)\,\mathrm{erfc}\,(\alpha\sqrt{\tau})]\,, \quad 0 \le \tau << 1\,. \tag{49}$$

If the materials of the strip and foundation are the same ($\varepsilon = 1$, $\lambda = 0$) or the Biot number $\mathrm{Bi} \to \infty$, from equality (49) it follows that the thermal contact of the bodies is close. It is seen from relations (45) and (46) that the temperatures of the strip and foundation with such a contact at small values of time can be calculated from the equations that represent the solutions of the thermal problem of friction for two semispaces [6]:

$$T_s^*(\zeta,\tau) \cong \frac{2\sqrt{\tau}}{1+\varepsilon}\,\mathrm{ierfc}\left(\frac{\zeta}{2\sqrt{\tau}}\right), \quad 0 \le \zeta < \infty\,; \quad T_f^*(\zeta,\tau) \cong \frac{2\sqrt{\tau}}{1+\varepsilon}\,\mathrm{ierfc}\left(\frac{|\zeta|}{2\sqrt{k^*\tau}}\right), \quad -\infty < \zeta \le 0\,.$$

The asymptotic (for large values of the parameter p) expressions for the Laplace trasnformants of the heat flux intensities have the form

$$\bar{q}_s^{\,*}(\zeta,p) \approx \frac{(\varepsilon\sqrt{p} + \mathrm{Bi})}{2\varepsilon p\,(\alpha + \sqrt{p})}\exp(-\zeta\sqrt{p})\,, \quad 0 \le \zeta \le 1\,;$$

(50)

$$\bar{q}_f^{\,*}(\zeta,p) \approx \frac{(\sqrt{p} + \mathrm{Bi})}{2p\,(\alpha + \sqrt{p})}\exp\left(\zeta\sqrt{\frac{p}{k^*}}\right), \quad -\infty < \zeta \le 0\,.$$

With the aid of the first equation of (44) and the coupling [16]

$$L^{-1}\left[\frac{\exp(-|\zeta|\sqrt{p})}{\sqrt{p}\,(\alpha + \sqrt{p})};\tau\right] = \exp\left(\alpha\,|\zeta| + \alpha^2\tau\right)\mathrm{erfc}\left(\frac{|\zeta|}{2\sqrt{\tau}} + \alpha\sqrt{\tau}\right)$$

from Eqs. (50) we find asymptotic (at small values of time) expressions for the dimensionless heat flux intensities in the strip and foundation:

$$q_s^*(\zeta,\tau) \approx \frac{1}{1+\varepsilon}\,\mathrm{erfc}\left(\frac{\zeta}{2\sqrt{\tau}}\right) - \frac{\lambda}{2}\exp\left(\alpha\,|\zeta| + \alpha^2\tau\right)\mathrm{erfc}\left(\frac{\zeta}{2\sqrt{\tau}} + \alpha\sqrt{\tau}\right), \quad 0 \le \zeta \le 1\,, \quad 0 < \tau << 1\,; \tag{51}$$

$$q_f^*(\zeta,\tau) \approx \frac{\varepsilon}{1+\varepsilon}\,\mathrm{erfc}\left(\frac{|\zeta|}{2\sqrt{k^*\tau}}\right) + \frac{\lambda}{2}\exp\left(\frac{\alpha|\zeta|}{\sqrt{k^*}} + \alpha^2\tau\right)\mathrm{erfc}\left(\frac{\zeta}{2\sqrt{k^*\tau}} + \alpha\sqrt{\tau}\right), \quad -\infty < \zeta \le 0\,, \quad 0 < \tau << 1\,. \tag{52}$$

On the contact surface $\zeta = 0$ Eqs. (51) and (52) yield

$$q_s^*(0,\tau) \approx \frac{1}{1+\varepsilon} - \frac{\lambda}{2}\exp(\alpha^2\tau)\,\mathrm{erfc}\,(\alpha\sqrt{\tau})\,, \quad q_f^*(0,\tau) \approx \frac{1}{1+\varepsilon} + \frac{\lambda}{2}\exp(\alpha^2\tau)\,\mathrm{erfc}\,(\alpha\sqrt{\tau})\,, \quad 0 < \tau << 1\,.$$

whence we obtain

$$q_f^*(0,\tau) + q_s^*(0,\tau) = 1\,, \quad q_f^*(0,\tau) - q_s^*(0,\tau) = -\lambda\,[1 - \exp(\alpha^2\tau)\,\mathrm{erfc}\,(\alpha\sqrt{\tau})]\,, \quad 0 < \tau << 1\,,$$

which, with account for the difference between the contact temperatures (49), indicates the fulfillment of the boundary thermal conditions (11) and (12).

Let us construct the asymptotics of the solutions of (22) and (23) at high values of the dimensionless time τ. The asymptotic expressions of the transforms (17)–(21) at small values of the parameter p of the Laplace integral transformation (16) have the form

$$\overline{T}_s^*(\zeta, p) \cong \frac{1-\zeta}{2+\mathrm{Bi}}\left[\frac{(2+\mathrm{Bi})\,\beta+\sqrt{p}}{p\,(\beta+\sqrt{p}\,)}\right], \quad 0 \le \zeta \le 1\,; \qquad \overline{T}_f^*(\zeta, p) \cong \frac{(1+\mathrm{Bi})\,\beta}{\mathrm{Bi}}\left[\frac{1+\zeta\sqrt{p/k^*}}{p\,(\beta+\sqrt{p}\,)}\right], \quad -\infty < \zeta \le 0\,, \tag{53}$$

where

$$\beta = \frac{\mathrm{Bi}}{\varepsilon\,(2+\mathrm{Bi})}\,. \tag{54}$$

With the aid of the transition formulas [16]

$$L^{-1}\left[\frac{1}{\sqrt{p}\,(\beta+\sqrt{p}\,)};\,\tau\right] = \exp(\beta^2\tau)\,\mathrm{erfc}\,(\beta\sqrt{\tau}\,)\,, \quad L^{-1}\left[\frac{1}{p\,(\beta+\sqrt{p}\,)};\,\tau\right] = 1 - \exp(\beta^2\tau)\,\mathrm{erfc}\,(\beta\sqrt{\tau}\,)$$

from relations (53) we obtain asymptotic expressions for the dimensionless temperatures in the strip and foundation:

$$T_s^*(\zeta, \tau) \cong (1-\zeta)\left[1 - \frac{1+\mathrm{Bi}}{2+\mathrm{Bi}}\exp(\beta^2\tau)\,\mathrm{erfc}\,(\beta\sqrt{\tau}\,)\right], \quad 0 \le \zeta \le 1\,, \quad \tau >> 1\,; \tag{55}$$

$$T_f^*(\zeta, \tau) \cong \frac{1+\mathrm{Bi}}{\mathrm{Bi}}\left[1 - \left(1 - \beta\frac{\zeta}{\sqrt{k^*}}\right)\exp(\beta^2\tau)\,\mathrm{erfc}\,(\beta\sqrt{\tau}\,)\right], \quad -\infty < \zeta \le 0\,, \quad \tau >> 1\,. \tag{56}$$

From Eqs. (55) and (56), subject to the notation of (54), we find the temperature jump on the contact surface:

$$T_s^*(0, \tau) - T_f^*(0, \tau) = -\frac{1}{\mathrm{Bi}}\left[1 - 2\,\frac{1+\mathrm{Bi}}{2+\mathrm{Bi}}\exp(\alpha^2\tau)\,\mathrm{erfc}\,(\beta\sqrt{\tau}\,)\right], \quad \tau >> 1\,, \tag{57}$$

and the asymptotics (at $\tau >> 1$) of the temperatures in the case of an ideal thermal contact between the strip and foundation ($\mathrm{Bi} \to \infty$):

$$\begin{aligned} T_s^*(\zeta, \tau) &\cong (1-\zeta)\left[1 - \exp\left(\frac{\sqrt{\tau}}{\varepsilon}\right)^2 \mathrm{erfc}\left(\frac{\sqrt{\tau}}{\varepsilon}\right)\right], \quad 0 \le \zeta \le 1\,; \\ T_f^*(\zeta, \tau) &\cong 1 - \left(1 - \frac{\zeta}{\varepsilon\sqrt{k^*}}\right)\exp\left(\frac{\sqrt{\tau}}{\varepsilon}\right)^2 \mathrm{erfc}\left(\frac{\sqrt{\tau}}{\varepsilon}\right), \quad -\infty < \zeta \le 0\,. \end{aligned} \tag{58}$$

The corresponding asymptotic expressions for the heat flux intensities have the form

$$q_s^*(\zeta, \tau) \cong 1 - \frac{1+\mathrm{Bi}}{2+\mathrm{Bi}}\exp(\beta^2\tau)\,\mathrm{erfc}\,(\beta\sqrt{\tau}\,)\,, \quad 0 \le \zeta \le 1\,, \quad \tau >> 1\,; \tag{59}$$

$$q_f^*(\zeta, \tau) \cong \frac{1+\mathrm{Bi}}{2+\mathrm{Bi}}\left[\frac{\zeta}{\sqrt{\pi k^*\tau}} + \left(1 - \beta\frac{\zeta}{\sqrt{k^*}}\right)\exp(\beta^2\tau)\,\mathrm{erfc}\,(\beta\sqrt{\tau}\,)\right], \quad -\infty < \zeta \le 0\,, \quad \tau >> 1\,. \tag{60}$$

It follows from Eqs. (59) and (60) that the heat flux intensity is constant over the strip thickness, but in the foundation it changes linearly with distance from the contact surface. Moreover, from these very equations we find that

$$q_f^*(0, \tau) + q_s^*(0, \tau) = 1\,, \quad q_f^*(0, \tau) - q_s^*(0, \tau) = -1 + 2\,\frac{1+\mathrm{Bi}}{2+\mathrm{Bi}}\exp(\beta^2\tau)\,\mathrm{erfc}\,(\beta\sqrt{\tau}\,)\,, \quad \tau >> 1\,,$$

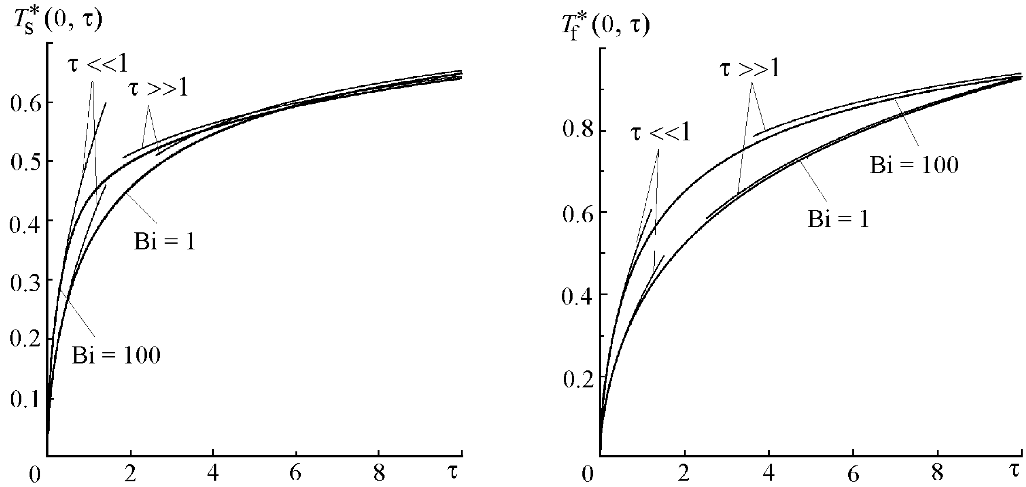

Fig. 2. Evolution of dimensionless temperatures on the frictional surface $\zeta = 0$ of the strip and foundation for two values of the Biot number Bi.

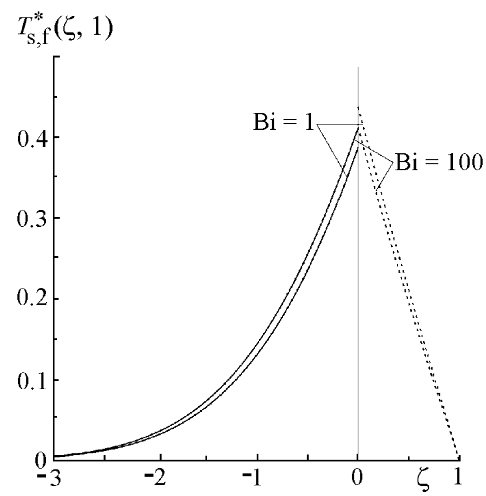

Fig. 3. Distribution of the dimensionless temperature in the strip ($0 \le \zeta \le 1$, dashed lines) and foundation ($-\infty \le \zeta \le 0$, solid lines) for a fixed Fourier number $\tau = 1$ and two Biot numbers Bi.

from which, subject to relation (57), it follows that the boundary conditions (11) and (12) are satisfied.

Numerical Analysis and Conclusions. The results for the dimensionless temperature $T_{s,f}^*$ and heat fluxes $q_{s,f}^*$ are presented in Figs. 2–4. The calculations were carried out for an FMC-11 metal-ceramic strip (K_s = 34.31 W/(m·K); $k_s = 15.2 \cdot 10^{-6}$ m^2/sec) and a foundation made from ChKhMK pig iron (K_f = 51 W/(m·K); $k_f = 14 \cdot 10^{-6}$ m^2/sec) [17]. Such a friction pair is used in the friction units of brakes [18]. The evolution of the dimensionless temperature $T_{s,f}^*$ on the contact surface $\zeta = 0$ for two values of the Biot number is shown in Fig. 2. The calculations were carried out using the equations of the exact solution of problem (22), (23), as well as the asymptotic solutions for low, (45), (46), and high, (55), (56), Fourier numbers. The time intervals of the application of these solutions are seen.

The change in the dimensionless temperature in the strip and foundation along the normal to the friction surface for a fixed value of the Fourier number $\tau = 1$ and two values of the Biot number, Bi = 1 and 100, is shown in Fig. 3. A maximum temperature is attained on the friction surface $\zeta = 0$. Note the presence of a temperature jump on this surface for Bi = 1 and its absence for Bi = 100. The decrease in the temperature in the strip from a maximum value on the contact surface to zero on the external surface occurs linearly over the thickness. The effective depth of

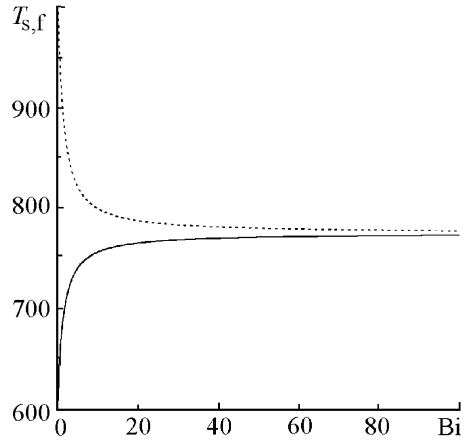

Fig. 4. Dependence of the temperatures of the strip (dashed line) and foundation (solid line) on the friction surface on the Biot number Bi at t = 1 sec. $T_{s,f}$, ^{o}C.

the heating of the foundation — the distance at which the temperature comprises 5% of the maximum one on the frictional surface — is equal to about three thicknesses of the strip.

The dependence of the temperatures of the strip and foundation on the frictional surface on the Biot number is shown in Fig. 4. An increase in the thermal conductivity (a decrease in thermal resistance) favors the equilibration of contact temperatures. The given data indicate that for values Bi ≥ 100 in calculations of temperature one may use the solution (37)–(40) in the case of an ideal thermal contact between the strip and foundation.

Conclusions. An analytical solution is obtained for the contact thermal problem of friction for the plane-parallel strip–foundation tribosystem in which heat transfer between the elements of the friction pair is taken into account. Convenient asymptotics of temperatures and heat flux intensities in the strip and foundation at low and high values of the Fourier number are given. The influence of the dimensionless thermal conductivity coefficient of contact (the Biot number) on change of the temperature in time and over the thickness from the friction surface is studied. The lower limit of the Biot number at which the thermal contact between the metal-ceramic strip and pig iron foundation can be considered complete has been established.

NOTATION

Bi, Biot number; d, strip thickness; erf (x), the Gaussian error function; erfc (x) = 1 − erf (x); ierfc (x) = $\pi^{-1/2}$ exp ($-x^2$) − x erfc (x); f, friction coefficient; h, thermal conductivity of the contact; K, thermal conductivity coefficient; k, thermal diffusivity coefficient; p_0, external pressure; q, specific power of friction; T, temperature; T_0, multiplier having the dimensionality of temperature; $T^* = T/T_0$, dimensionless temperature; t, time; V, sliding velocity; x, y, z, spatial variables; ζ, dimensionless variables; τ, Fourier number. Subscripts: f, foundation; s, strip.

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